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LETTER TO THE EDITOR

Viscous damping effect on the magnetic penetration depth in superconducting $ErNi_2B_2C$

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Abstract. The equation of motion of a fluxoid with viscous forces is solved to calculate the effective London penetration depth which is found to vary as the square root of the magnetic field. The London penetration depth as a function of magnetic field in single crystals of $ErNi_2B_2C$ is in reasonable agreement with the theory.

The London penetration depth is determined by a constant factor when the diamagnetic critical current is expressed in terms of the vector potential

$$\boldsymbol{J} = -\frac{c}{4\pi\lambda_L^2}\boldsymbol{A}. \tag{1}$$

In such an expression the wave vector dependence of the electron operators has not been considered. The inclusion of the wave vector dependence of the electrons and the suitable summation over such wave vectors requires the use of density of states [1]. Kogen *et al* [2] have derived the London equations from the microscopic theory which show that the inverse square of the London penetration depth depends on the density of states, per spin, at the Fermi level. Won and Maki [3] have found that the residual density of states varies as the square root of the magnetic field, so that we can conclude that λ_L varies as the inverse one-quarter power of the magnetic field, $\lambda_L \propto B^{-1/4}$. Yip and Sauls [4] and Xu *et al* [5] have considered a nonlinear current–velocity equation from which they argued that a pair-breaking effect reduces the effective superfluid density, and hence penetration depth increases quadratically with the magnetic field, $\lambda_L \propto B^2$. For a field parallel to a node, the effective penetration length is predicted to be proportional to the field, $\lambda_{eff} \propto B$. Affleck *et al* [6] have generalized the London model starting from a Ginzburg–Landau free energy density to consider the coordinate dependence of the order parameter which explains the flux-lattice structures. This work [7] can be interpreted to give rise to a wave vector dependence, and hence temperature and field dependence, of the effective penetration depth. Kosztin and Leggett [8] have shown that non-local electrodynamics gives rise to a T^2 dependence in the London penetration depth. It has been reported by Eskildsen *et al* [9] that rf kinetic inductance as measured in a tunnel diode changes the resonant frequency, due to changes in the penetration depth. Such a measurement in $ErNi_2B_2C$ shows $\lambda_{eff} \sim \sqrt{B}$, which is not understood theoretically.

In this letter, we show that fluxoid viscous damping gives rise to an effective London penetration depth which at large fields varies as the square root of the magnetic field. The calculation is in accord with recent neutron reflectivity and inductance measurements of the penetration depth in $ErNi_2B_2C$ as a function of magnetic field up to 2 kOe.

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We assume that fluxoids are subject to a harmonic force with force constant k, viscous force ηv proportional to the velocity v and ordinary fluxoid mass M, multiplied by the acceleration, determined by the quantized flux,

$$M\frac{\mathrm{d}v}{\mathrm{d}t} + \eta v + kx = \frac{1}{c}J\phi_0. \tag{2}$$

The fluxoid velocity has the time dependence of $v = v_0\,\mathrm{e}^{-\mathrm{i}\omega t}$ which substituted in (1) gives

$$v = \frac{J\phi_o}{c\{\eta - \mathrm{i}\omega M + (\mathrm{i}k/\omega)\}}. \tag{3}$$

The moving fluxoid produces an electric field, $E_\psi = -(1/c)vB$, that opposes the current, so that

$$E_\phi = -\frac{J\phi_0 B}{c^2\{\eta - \mathrm{i}\omega M + (\mathrm{i}k/\omega)\}}. \tag{4}$$

Differentiating (1) with respect to time and replacing the time derivative of the vector potential, $-\partial A/\partial t = E + \nabla\phi$, we can write

$$\frac{\mathrm{d}J}{\mathrm{d}t} = \frac{c^2}{4\pi\lambda_L^2}\big(E + E_\phi\big). \tag{5}$$

Substituting (4) into (5) and taking the time dependence of the current as $J = J_0\,\mathrm{e}^{\mathrm{i}\omega t}$, we find

$$J = E\left[\frac{\phi_0 B}{c^2\{\eta - \mathrm{i}\omega M + (\mathrm{i}k/\omega)\}} - \frac{4\pi\mathrm{i}\omega\lambda_L^2}{c^2}\right]^{-1} \tag{6}$$

so that the complex resistivity may be defined by

$$\rho^{-1} = \frac{J}{E} = \left[\frac{\phi_0 B}{c^2\{\eta - \mathrm{i}\omega M + \mathrm{i}k/\omega\}} - \frac{4\pi\mathrm{i}\omega\lambda_L^2}{c^2}\right]^{-1}. \tag{7}$$

We define an effective value of the penetration depth, λ_{eff} at an effective ac frequency ω_{eff}, such that

$$\lambda_{eff}^2 = \lambda_L^2(0)\left[1 + \mathrm{i}\frac{\phi_0 B}{4\pi\omega\{\eta - \mathrm{i}\omega M + \mathrm{i}k/\omega\}\lambda_L^2(0)}\right]. \tag{8}$$

We define the mass term, $m = (k/\omega) - \omega M$, so that

$$\lambda_{eff} = \lambda_L(0)\left[1 + \frac{\phi_0 B}{4\pi\omega\lambda_L^2(0)(m - \mathrm{i}\eta)}\right]^{1/2} \tag{9}$$

in which the complexity is caused by the application of the magnetic field so that there is a change in the effective London penetration depth due to the magnetic field. We separate the real and imaginary parts to define the absolute value of the London penetration depth as

$$\lambda_{eff} = \lambda_L(0)\left[\left\{1 + \frac{\phi_0 Bm}{4\pi\omega\lambda_L^2(0)(m^2 + \eta^2)}\right\}^2 + \left\{\frac{\phi_0 B\eta}{4\pi\omega\lambda_L^2(m^2 + \eta^2)}\right\}^2\right]^{1/4}. \tag{10}$$

At large magnetic fields the quantity 1 in the first term on the right-hand side is small compared with the second term so that

$$\lambda_{eff} \approx B^{1/2}\big(m^2 + \eta^2\big)^{-1/4}\left[\frac{\phi_0}{4\pi\omega}\right]^{1/2} \tag{11}$$

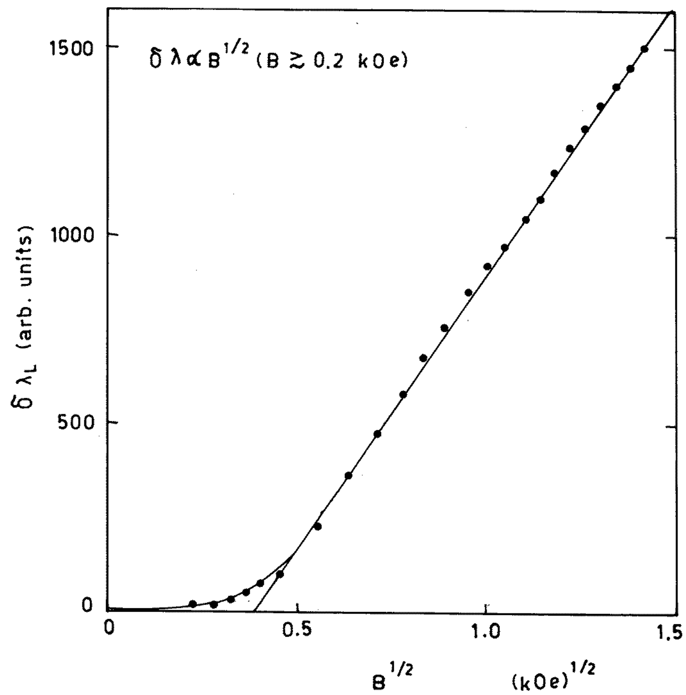

Figure 1. Change in the London penetration depth of single crystals of $ErNi_2B_2C$ as a function of the square root of the magnetic field. The dots have been drawn on the basis of experimental measurements performed by Eskildsen *et al* [9]. The straight line showing $\delta\lambda_L \propto B^{1/2}$ is based on the theory given in the text.

which predicts that at large fields the effective London penetration depth varies as the square root of the magnetic field. It has been previously found [10] that the surface resistance and reactance increase with magnetic field as a result of energy loss through fluxoids driven by superconducting currents. The change in the London penetration depth is consistent with the change in reactance upon application of the magnetic field.

The change in the London penetration depth, $\delta\lambda_L$, as a function of magnetic field, $B \parallel c$ and $H_{\text{rf}} \perp c$, for $T = 4.96$ K, in the single crystals of $ErNi_2B_2C$ has been deduced by rf kinetic inductance measurements by Eskildsen *et al* [9] who also found a topological transition in the flux lattice using small-angle neutron scattering. From a plot of the logarithm of the form-factor versus the magnetic field, it is found that $\lambda \approx 500$ Å and coherence length, $\xi = 135$ Å at $T = 2.2$ K. We have extracted the values of $\delta\lambda_L$ from their measurements and plotted them in figure 1 as a function of $B^{1/2}$. It is clear that there are two separate mechanisms: one at low magnetic fields, $B < 0.2$ kOe, where the field dependence of $\delta\lambda_L$ is a slowly varying function of magnetic field, and another at high magnetic fields, $B > 0.2$ kOe, at which the measured values are proportional to the square root of the field. The penetration depth as well as the coherence length has also been measured [11] in YNi_2B_2C and $LuNi_2B_2C$ single crystals with $\lambda_L = 1060 \pm 30$ Å for both the compounds, and $\xi = 88 \pm 3$ Å for the Y and 82 ± 2 Å for the Lu compound. The properties of the Y compound are also reported by Tomy *et al* [12]. It is obvious that at high fields the measured values are in good agreement with those predicted by the theory of (11) based on the viscous damping of vortices in superconductors. The pinning force constant is reduced to $k - \omega^2 M$ due to kinetic motion of vortices. Thus the observation of the variation of the penetration depth as a function of square root of field shows that the flux is moving in a medium of pinning forces damped by the viscous motion.

Usually the London penetration depth determines the distance up to which the magnetic field enters the superconductor according to exponential decay, $B = B_0 \mathrm{e}^{-x/\lambda_L}$. This penetration depth is independent of temperature and field. However, due to wave vector space, a small temperature dependence is introduced which depends on the symmetry of the gap [1]. The boson nature of vortices giving rise to quantum effects in vortices has also been reported [13]. The electromagnetic contribution to the penetration depth also depends on temperature [8]. In the present work, we have shown that the effective penetration depth varies as the square root of the magnetic field due to the viscous forces on the fluxoids.

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